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GAUGED MATTER COUPLING IN $N=4$ SUPERGRAVITY

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Gauged $N=4$ supergravity with an arbitrary number of matter multiplets is constructed from a superconformal starting point. It includes both the $SO(4)$ and $SU(4)$ symmetric $N=4$ supergravity theories, and all their gaugings. Noncompact Yang–Mills symmetries may mix the matter and supergravity vector fields. We establish that in matter coupled $N=4$ supergravity theories the super-Higgs effect can occur with a vanishing cosmological constant. An example is given with gauged $SO(3) \times SO(2, 1)$ symmetry in which the scalar potential vanishes completely, and all four supersymmetries are broken.

1. Introduction

$N=4$ supergravity [1–4] has many interesting properties which justify its investigation. It is the lowest N theory with fundamental physical scalar fields. In the supergravity action these scalar fields occur nonpolynomially in a way determined by a noncompact $SU(1, 1) \times U(1)$ symmetry [4]. In the gauged supergravity theory they give rise to a scalar potential, which implies the possibility of spontaneous supersymmetry breaking in a pure supergravity theory. $N=4$ supergravity is the highest N theory for which a superconformal theory is available [5]. Conformal supergravity is helpful in the systematic construction of matter couplings, but also deserves consideration as a possible fundamental theory*.

In this paper we continue the investigation of matter coupling in $N=4$ supergravity. The basic ingredient is the $N=4$ vector multiplet [7] in a superconformal background, for which the complete lagrangian and transformation rules were recently obtained [8]. It was shown in [8] that the transition to $N=4$ Poincaré supergravity is possible only if at least six vector multiplets are employed. If this number is precisely six, one obtains the globally $SU(4)$ symmetric version of $N=4$ supergravity [4], additional vector multiplets are coupled to this theory. The vector fields have abelian gauge transformations, but these can be extended [9] to non-abelian transformations if all the fields of the vector multiplets transform according to the adjoint representation of a Yang–Mills group G . With six multiplets one obtains in the nonabelian case the gauged $N=4$ supergravity theory of Freedman and Schwarz [10]. The corresponding potential has no stationary points.

* For a recent review of superconformal methods and prospects, see [6].

$N = 4$ supergravity was originally constructed [1] with a global $SO(4)$ symmetry of the action, although it was soon realized that the equations of motion allow a larger symmetry [2]. The two versions [$SO(4)$ and $SU(4)$] of $N = 4$ supergravity are related by a duality transformation on three of the six vector fields [4], which changes the character of these fields under parity transformations. Classically the two theories are equivalent. The gauging of a global internal symmetry breaks the symmetry of the equations of motion, and different starting points [$SO(4)$ or $SU(4)$] yield inequivalent theories. Indeed, the scalar potential of gauged $SO(4)$ supergravity [11] has a different structure from that of [10], and does have a stationary point. Recently, a third inequivalent version of gauged $N = 4$ supergravity was constructed with positive cosmological constant and broken supersymmetry [12].

Our aim in this paper is to extend previous work [8,9] to include also the $SO(4)$ -type supergravity theories and their coupling to matter. As it turns out, this can be done in a quite natural way in the superconformal framework by relaxing certain conditions that were implicit in [8]. The scalar fields of the Weyl multiplet parametrize the coset space $SU(1, 1)/U(1)$ [5], and transform under global $SU(1, 1)$. The vector multiplet in a superconformal background has an $SU(1, 1)$ symmetry of the equations of motion. In [8] the two $SU(1, 1)$ groups were identified, but it is in fact possible to admit other isomorphisms, involving an arbitrary $SU(1, 1)$ element, between these groups. The corresponding changes in the superconformal action can be implemented by a duality transformation. If several vector multiplets are present, the isomorphisms (duality rotations) may be chosen differently for each multiplet. Thus one can introduce additional parameters, whose (relative) values determine the global symmetry of the resulting Poincaré theory.

In sect. 2 we present the ingredients for the construction of duality rotated theories. These are used in sect. 3 to obtain the main result of this paper: the complete lagrangian for an arbitrary number of $N = 4$ Yang–Mills multiplets coupled to $N = 4$ supergravity. This theory has remnants of the superconformal symmetry groups, in particular local $U(4)$ symmetry. The parameters corresponding to the duality orientations determine the global symmetry of the superconformal action. The action has the local internal symmetry of the Yang–Mills multiplets which make up the superconformal action. In the Poincaré theory this internal symmetry may mix vector fields of $N = 4$ supergravity with the $N = 4$ matter multiplets. Since the local gauge group must be a subgroup of the global symmetry group, there is a relation between the duality orientations and the allowed gauge groups. To illustrate the result of sect. 3, we recover the known results about the three different versions of gauged $N = 4$ supergravity [10–12] at the end of that section.

The local gauge groups may involve noncompact factors, and we present in sect. 4 the complete list of simple noncompact gauge groups which can occur in the context of $N = 4$ supergravity. The analysis of Poincaré supergravity coupled to n matter multiplets is complicated by the fact that the scalar and some of the spin- $\frac{1}{2}$ fields are subject to a constraint equation. The scalar fields parametrize the manifold

$[\text{SO}(6, n)/\text{SO}(6) \times \text{SO}(n)] \times [\text{SU}(1, 1)/\text{U}(1)]$, for which it is difficult to obtain a convenient coordinate system. We analyze the case of seven vector multiplets which, in the Poincaré context, corresponds to supergravity coupled to a single vector multiplet. One may gauge $\text{SO}(3) \times \text{SO}(3)$, and in that case the potential and its properties are similar to those of gauged $N = 4$ supergravity. The second possibility is to gauge $\text{SO}(3) \times \text{SO}(2, 1)$, and then the result is quite different. We find that the scalar potential does not have stationary points, unless the parameters are chosen such that the potential vanishes completely. Thus we establish that flat scalar potentials, which exist in matter coupling in $N = 1$ [13] and $N = 2$ [14] supergravity, are also possible in $N = 4$ supergravity. In our example all four supersymmetries are broken for arbitrary values of the scalar fields.

2. Duality transformations

The basic ingredients of $N = 4$ supergravity theories are the $N = 4$ vector multiplet [7] and the $N = 4$ superconformal gauge (Weyl) multiplet [5]. In [8] it was shown that six abelian vector multiplets coupled to conformal supergravity yield the $\text{SU}(4)$ symmetric version [4] of $N = 4$ Poincaré supergravity. In this section we discuss how duality transformations can be used to construct the $\text{SO}(4)$ symmetric version [1] of $N = 4$ supergravity by superconformal methods. To set the stage for the construction of duality rotated theories we retrace in part the analysis of [8].

The $N = 4$ vector multiplet contains fields A_μ , $\psi^i (\gamma^5 \psi^i = \psi^i, \psi_i \equiv (\psi^i)^*)$ and $\phi_{ij} (\phi_{ij} = -\phi_{ji}, \phi^{ij} \equiv (\phi_{ij})^* = -\frac{1}{2} \varepsilon^{ijkl} \phi_{kl})$, with spins $1, \frac{1}{2}$ and 0 respectively. The fermions and scalars transform under the 4- and 6-dimensional representations of $\text{SU}(4)$ respectively, while A_μ is an $\text{SU}(4)$ singlet*. The supersymmetric action in flat space reads

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^+ F^{\mu\nu+} - \frac{1}{2} \bar{\psi}^i \not{\partial} \psi_i - \frac{1}{4} (\partial_\mu \phi_{ij}) (\partial^\mu \phi^{ij}) + \text{h.c.}, \quad (2.1)$$

where

$$F_{\mu\nu}^\pm = \frac{1}{2} (F_{\mu\nu} \pm \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}), \quad (F_{\mu\nu}^\pm)^* = F_{\mu\nu}^\mp. \quad (2.2)$$

The vector multiplet can be coupled to conformal supergravity, and the resulting superconformal invariant which generalizes (2.1) was presented in [8]. Here we concentrate on the terms involving the field strength $F_{\mu\nu}$. The corresponding equations of motion and Bianchi identities have an $\text{SU}(1, 1)$ duality symmetry. In terms of the tensor

$$G_{\mu\nu}^+ = -\frac{2}{e} \frac{\partial \mathcal{L}}{\partial F^{\mu\nu+}}, \quad (2.3)$$

the equations of motion and Bianchi identities read

$$\begin{aligned} \partial_\mu (e (G^{\mu\nu+} + G^{\mu\nu-})) &= 0, \\ \partial_\mu (e (F^{\mu\nu+} - F^{\mu\nu-})) &= 0. \end{aligned} \quad (2.4)$$

* We use throughout this paper the conventions of refs. [5, 8].

To establish the $SU(1, 1)$ symmetry of these equations it is convenient to use the combinations

$$F_{1\mu\nu}^+ = \frac{1}{2}(G_{\mu\nu}^+ - F_{\mu\nu}^+), \quad F_{2\mu\nu}^+ = \frac{1}{2}(G_{\mu\nu}^+ + F_{\mu\nu}^+). \quad (2.5)$$

Eqs. (2.4) are equivalent to

$$\partial_\mu \left[e \begin{pmatrix} F_1^{+\mu\nu} \\ F_2^{+\mu\nu} \end{pmatrix} + e\sigma_1 \begin{pmatrix} F_1^{+\mu\nu} \\ F_2^{+\mu\nu} \end{pmatrix}^* \right] = 0. \quad (2.6)$$

Under linear transformations

$$\begin{pmatrix} F_{1\mu\nu}^+ \\ F_{2\mu\nu}^+ \end{pmatrix} \rightarrow \begin{pmatrix} F_{1\mu\nu}^+ \\ F_{2\mu\nu}^+ \end{pmatrix}' = C \begin{pmatrix} F_{1\mu\nu}^+ \\ F_{2\mu\nu}^+ \end{pmatrix} \quad (2.7)$$

(2.6) is preserved if C satisfies

$$C^{-1}\sigma_1 C^* = \sigma_1. \quad (2.8)$$

This condition is fulfilled for $C \in SU(1, 1)$. However, $G_{\mu\nu}^+$ and $F_{\mu\nu}^+$ are related by (2.3), and this imposes restrictions on the form of the action. Since we are considering symmetries of the equations of motion, the on-shell action must be invariant under $SU(1, 1)$. This implies that \mathcal{L} can be written in the form [15]

$$\mathcal{L} = -\frac{1}{4}eF_{\mu\nu}^+ G^{\mu\nu+} + \text{h.c.} + \mathcal{L}_{\text{inv}}, \quad (2.9)$$

where \mathcal{L}_{inv} is $SU(1, 1)$ invariant. To construct $SU(1, 1)$ invariants which contain $F_{\mu\nu}^+$ one requires additional fields which transform non-trivially under $SU(1, 1)$. The $N=4$ Weyl multiplet contains an $SU(1, 1)$ doublet of scalar fields ϕ_α ($\alpha = 1, 2$) satisfying $(\phi^1 \equiv (\phi_1)^*, \phi^2 \equiv -(\phi_2)^*)$

$$\phi^\alpha \phi_\alpha = 1. \quad (2.10)$$

Under the local chiral $U(1)$ transformations of $N=4$ conformal supergravity ϕ_α transforms with weight $c = -1$. With ϕ_α one can construct an $SU(1, 1)$ element U ,

$$U = \begin{pmatrix} \phi_1 & -\phi^2 \\ \phi_2 & \phi^1 \end{pmatrix}, \quad (2.11)$$

which transforms under $SU(1, 1)$ as

$$U \rightarrow U' = CU. \quad (2.12)$$

Then

$$\begin{pmatrix} \bar{G}_{\mu\nu}^+ \\ \bar{F}_{\mu\nu}^+ \end{pmatrix} \equiv U^{-1} \begin{pmatrix} F_{1\mu\nu}^+ \\ F_{2\mu\nu}^+ \end{pmatrix} \quad (2.13)$$

defines two $SU(1, 1)$ invariants. Using these invariants, the only $U(1)$ invariant action of the form (2.9) is

$$\mathcal{L} = -\frac{1}{4}eF_{\mu\nu}^+G^{\mu\nu+} - \frac{1}{2}ce\bar{F}_{\mu\nu}^+\bar{G}^{\mu\nu+} + \text{h.c.}, \quad (2.14)$$

where c is a constant. The requirement (2.3) implies (up to an ambiguity discussed below) $c = 1$ and

$$\begin{aligned} G_{\mu\nu}^+ &= \frac{\phi^1 - \phi^2}{\Phi} F_{\mu\nu}^+ + \frac{2}{\Phi} H_{\mu\nu}^+, \\ \bar{G}_{\mu\nu}^+ &= H_{\mu\nu}^+, \\ \bar{F}_{\mu\nu}^+ &= \frac{1}{\Phi} F_{\mu\nu}^+ + \frac{\Phi^*}{\Phi} H_{\mu\nu}^+, \end{aligned} \quad (2.15)$$

with

$$\Phi = \phi^1 + \phi^2. \quad (2.16)$$

Here $H_{\mu\nu}^+$ is independent of $F_{\mu\nu}^+$ but otherwise undetermined at this stage. There is another solution for $c = -1$ in which it is $\bar{G}_{\mu\nu}^+$ which depends on $F_{\mu\nu}^+$. In the context of conformal supergravity that solution can be rejected, because it is inconsistent with the chiral $U(1)$ symmetry of the superconformal multiplet.

Under a duality transformation the action (2.14) and the relations (2.15) retain the same form, with $G_{\mu\nu}^+$, $F_{\mu\nu}^+$ and ϕ_α replaced by $G_{\mu\nu}^{\prime+}$, $F_{\mu\nu}^{\prime+}$ and ϕ'_α , where primed quantities are obtained from (2.7) and (2.12). Thus the fields $F_{\mu\nu}^{\prime+}$ couple to scalars ϕ'_α , which are related to the ϕ_α of the superconformal multiplet by an $SU(1, 1)$ transformation. Since ϕ_α are the only fields of the Weyl multiplet which transform under $SU(1, 1)$, this transformation preserves the superconformal symmetry of the matter coupling. When several vector multiplets are coupled to conformal supergravity, the superconformal invariant is just a sum of the invariants for the different multiplets [8]. The multiplets mix after the transition to the Poincaré gauge and the elimination of the auxiliary fields. When several vector multiplets are coupled to conformal supergravity using different duality orientations (i.e. using scalars ϕ'_α instead of ϕ_α) for different multiplets, then these $SU(1, 1)$ orientations show up as parameters in the resulting Poincaré theory. In this way the difference between the $SU(4)$ and $SO(4)$ versions of $N = 4$ supergravity can be understood at the superconformal level.

Let us consider in more detail the effect of replacing ϕ_α by ϕ'_α . It is convenient to write the $SU(1, 1)$ matrix C in the form

$$C = \begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix}; \quad \begin{aligned} a &= \frac{1}{2} e^{-i\alpha} \left(\frac{1+s^2}{s} - it \right), \\ b &= \frac{1}{2} e^{i\alpha} \left(\frac{1-s^2}{s} + it \right), \end{aligned} \quad (2.17)$$

where s and t parametrize the two noncompact directions in $SU(1, 1)$. On substituting ϕ'_α in (2.15), the action (2.14) takes on the form $(\Phi' = s(e^{i\alpha}\phi^1 + e^{-i\alpha}\phi^2))$

$$\mathcal{L}' = -\frac{1}{4}eF_{\mu\nu}^+F^{\mu\nu+}\left(\frac{e^{i\alpha}\phi^1 - e^{-i\alpha}\phi^2}{\Phi'_s} + i\frac{t}{s}\right) - e\frac{1}{\Phi'}F_{\mu\nu}^+H^{\mu\nu+} - \frac{1}{2}e\frac{\Phi'^*}{\Phi'}H_{\mu\nu}^+H^{\mu\nu+} + \text{h.c.} \quad (2.18)$$

The parameter t only appears in the first term, and adds a total derivative to the lagrangian density. We limit ourselves to $t=0$. Then the parameter s rescales the field strengths, and can be absorbed by a redefinition (in gauged supergravity theories a change in the coupling constant is required as well). Thus the only remaining parameter is the phase α . The precise effect of such phases on the resulting Poincaré theory can be deduced only after elimination of auxiliary fields, and this will be discussed in detail in the following sections.

The $SU(1, 1)$ rotated actions may also be obtained in the following way. In (2.11)–(2.13) the superconformal $SU(1, 1)$ group and the duality $SU(1, 1)$ group were identified. However, it is possible to relate them by an isomorphism of the form

$$C' = A^{-1}CA, \quad (2.19)$$

where $A \in SU(1, 1)$. Then ϕ_α transforms as $\phi'_\alpha = (C'\phi)_\alpha$, and (2.13) still defines $SU(1, 1)$ invariants if U is replaced by AU . This replacement of ϕ_α by $(A\phi)_\alpha$ in (2.14)–(2.15) leads immediately to the $SU(1, 1)$ rotated action (2.18).

With six vector multiplets, each with $\alpha=0$, the $SU(4)$ symmetric version of $N=4$ Poincaré supergravity can be constructed [8]. It contains 3 vectors and 3 axial vectors. The $SO(4)$ symmetric theory contains 6 vectors, and may be obtained from the $SU(4)$ symmetric Poincaré theory by a duality transformation [4]. In the superconformal context the relation between these two theories is a duality transformation of the type discussed above. Using again 6 vector multiplets, but now 3 with $\alpha=0$ and 3 with $\alpha=\frac{1}{2}\pi$, the $SO(4)$ Poincaré theory is obtained by the same procedure as in [8].

3. Gauged $N=4$ supergravity with matter

The action of an $N=4$ vector multiplet coupled to $N=4$ conformal supergravity contains, besides the superconformal gauge fields, also a number of matter fields which belong to the Weyl multiplet. These are scalars ϕ_α , which we already discussed in sect. 2, spin- $\frac{1}{2}$ fields Λ_i and $\chi^{\dot{j}}_k$ (in 4- and 20-dimensional representations of $SU(4)$, respectively), and the bosons $T_{ab}^{\dot{i}\dot{j}}$ ($T_{ab}^{\dot{i}\dot{j}} = -\frac{1}{2}\varepsilon_{abcd}T_{cd}^{\dot{i}\dot{j}} = -T_{ab}^{\dot{j}\dot{i}}$), E_{ij} ($E_{ij} = E_{ji}$) and $D^{\dot{i}\dot{j}}_{kl}$ (in a 20 of $SU(4)$). The fields ϕ_α and Λ_i are physical fields of the Poincaré theory, while E_{ij} and $T_{ab}^{\dot{i}\dot{j}}$ are auxiliary. The remaining fields $\chi^{\dot{j}}_k$ and $D^{\dot{i}\dot{j}}_{kl}$ play a special role. They occur linearly in the action, and their equation of motion therefore implies constraints on the fields of the vector multiplet.

The leading terms in the Poincaré action of an arbitrary number of vector multiplets coupled to supergravity without duality rotations, take the form [8]

$$\begin{aligned}
 e^{-1} \mathcal{L} = & \eta_{IJ} \left\{ -\frac{1}{4} F_{\mu\nu}^{+I} F^{\mu\nu+J} \frac{\phi^1 - \phi^2}{\Phi} - \frac{1}{2} \bar{\psi}^{Ii} \gamma^\mu \mathcal{D}_\mu \psi_i^J \right. \\
 & - \frac{1}{4} \mathcal{D}_\mu \phi_{ij}^I \mathcal{D}^\mu \phi^{ijJ} - \frac{1}{\Phi} \hat{F}_{\mu\nu}^{+I} K^{\mu\nu J+} - \frac{1}{4} E^{kl} \bar{\psi}_k^I \psi_l^J \\
 & \left. + \frac{1}{4} \varepsilon^{ijkl} \bar{\psi}_i^I \sigma \cdot T_{jk} \psi_l^J - \frac{1}{2} \frac{\Phi^*}{\Phi} K_{\mu\nu}^{+I} K^{\mu\nu+J} \right\} \\
 & - \frac{1}{4} R(\omega) - \frac{1}{2} e^{-1} \varepsilon^{\mu\nu\lambda\rho} \bar{\psi}_\mu^i \gamma_\nu \mathcal{D}_\lambda \psi_{\rho i} \\
 & + \frac{1}{8} (E^{kl} E_{kl} + 4 D_\alpha \phi^\alpha D_\alpha \phi_\alpha - 2 \bar{\Lambda}^k \gamma^\mu \mathcal{D}_\mu \Lambda_k) + \text{h.c.} \\
 & + \text{further } \psi_\mu - \text{dep. terms.} \tag{3.1}
 \end{aligned}$$

In (3.1) the fields χ_{ij}^k , D_{kl}^{ij} have already been eliminated. The resulting constraints are

$$\eta_{IJ} \phi_{ij}^I \phi^{kJ} = -\frac{1}{2} \delta_{[i}^k \delta_{j]}^I, \tag{3.2}$$

$$\eta_{IJ} \phi_{ij}^I \psi^{kJ} = 0. \tag{3.3}$$

The metric η_{IJ} is of the form ($I, J = 1, \dots, P$)

$$\eta_{IJ} = \text{diag}(-1, -1, -1, -1, -1, -1, +1, \dots, +1). \tag{3.4}$$

The constraint (3.2) implies that $P \geq 6$, and that at least 6 minus signs are present in η . The other $n = P - 6$ signs are then taken positive to ensure the correct sign for the kinetic terms. The terms in (3.1) containing $F_{\mu\nu}^{+I}$ are of the form discussed in sect. 2. The combination $K_{\mu\nu}^{+I}$ is given by

$$K_{\mu\nu}^{+I} = -\frac{1}{2} \bar{\Lambda}^i \sigma_{\mu\nu} \psi_i^I + T_{\mu\nu ij} \phi^{Iij}. \tag{3.5}$$

The derivatives \mathcal{D}_μ in (3.1) contain SU(4) and U(1) covariantizations, while D_μ , and tensors with a caret, are fully covariant. The ψ_μ -dependent terms not written explicitly can all be found in [8].

Following the discussion in sect. 2 we now generalize the coupling of vector multiplets to $N = 4$ conformal supergravity by introducing SU(1, 1) orientations α_I for each multiplet I , $I = 1, \dots, P$. This is done by the substitution

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_{1(I)} \\ \phi_{2(I)} \end{pmatrix} \equiv \begin{pmatrix} e^{-i\alpha_I} & 0 \\ 0 & e^{i\alpha_I} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{3.6}$$

in both the superconformal action and transformation rules of [8]. In the ungauged version of the Poincaré theory, i.e. in (3.1), this results in the replacement of ϕ_α by $\phi_{\alpha(I)}$. The action (3.1) had a global SO(6, n) symmetry, which is in general destroyed by the introduction of the phases α_I . This is relevant for the possibility of gauging subgroups of the original SO(6, n) group, to which we now turn.

If a local symmetry group G is introduced with A_μ^I as gauge fields, all fields of the vector multiplets must transform according to the adjoint representation. Invariance of the action then implies that η_{IJ} must be an invariant tensor:

$$f_{IJ}^K \eta_{KL} + f_{IL}^K \eta_{JK} = 0, \quad (3.7)$$

where f_{IJ}^K are the structure constants of G . Clearly the form of η (3.4) imposes a severe restriction on the noncompact part of the group G . This will be discussed further in sect. 4. The choice of the group G must be consistent with the choice of the additional parameters α_I . For each simple subgroup H of G , with gauge fields A_μ^I , the corresponding α_I must be the same. This follows from the invariance of the action under H -transformations, and can also be derived from the requirement of closure of the supersymmetry algebra.

With these restrictions in mind we present the generalization of the results of [9]. The action is:

$$\begin{aligned} e^{-1} \mathcal{L} = & \eta_{IJ} \left\{ -\frac{1}{4} F_{\mu\nu}^{+I} F^{\mu\nu+J} \frac{1}{\Phi_{(I)}} (\phi_{(I)}^1 - \phi_{(I)}^2) - \frac{1}{2} \bar{\psi}^{II} \gamma^\mu \mathcal{D}_\mu \psi_I^J \right. \\ & - \frac{1}{4} \mathcal{D}_\mu \phi_{ij}^I \mathcal{D}^\mu \phi^{ijJ} - \frac{1}{\Phi_{(I)}} \hat{F}_{\mu\nu}^{+I} K^{\mu\nu+J} - \frac{1}{4} E^{kl} \bar{\psi}_k^I \psi_l^J \\ & + \frac{1}{4} \epsilon^{ijkl} \bar{\psi}_i^I \sigma \cdot T_{jk} \psi_j^I - \frac{1}{2} \frac{\Phi_{(I)}^*}{\Phi_{(I)}} K_{\mu\nu}^{+I} K^{\mu\nu+J} \left. \right\} \\ & - \frac{1}{4} R(\omega) - \frac{1}{2} e^{-1} \epsilon^{\mu\nu\lambda\rho} \bar{\psi}_\mu^i \gamma_\nu \mathcal{D}_\lambda \psi_{\rho i} \\ & + \frac{1}{8} (E^{kl} E_{kl} + 4 D_a \phi^\alpha D_a \phi_\alpha - 2 \bar{\Lambda}^k \gamma^\mu \mathcal{D}_\mu \Lambda_k) \\ & + \eta_{IJ} \left\{ -\Phi_{(I)} \phi^{\dot{ij}} f_{KL}^J \bar{\psi}_i^K \psi_j^L + \Phi_{(I)} \bar{\Lambda}_k \psi^\mu W_i^{k\dot{J}} \right. \\ & + \frac{1}{4} |\Phi_{(I)}|^2 W_i^{j\dot{I}} W_j^{i\dot{J}} + \frac{1}{3} E^{kl} X_{kl}^{IJ} \Phi_{(I)}^* \\ & - \bar{\psi}^{II} \gamma^\mu \gamma^\nu \psi_\mu^j \mathcal{D}_\nu \phi_{ij}^J - \Phi_{(I)} \bar{\psi}_\mu^k \gamma^\mu \psi_j^I W_k^{j\dot{J}} \\ & - \frac{1}{2 \Phi_{(I)}} F_{ab}^{+I} (\bar{\psi}_{i\mu} \sigma_{ab} \gamma^\mu \psi^{i\dot{J}} - \bar{\psi}_\mu^i \gamma^\mu \sigma_{ab} \Lambda^j \phi_{ij}^I + 2 \bar{\psi}_{a\dot{i}} \psi_{b\dot{j}} \phi^{Jij}) \\ & + \frac{1}{4} e^{-1} \epsilon^{\mu\nu\lambda\rho} \bar{\psi}_{\mu k} \gamma_\nu \psi_\lambda^j \phi^{ikI} \tilde{\mathcal{D}}_\rho \phi_{ij}^I \\ & + \frac{1}{3} \Phi_{(I)} \bar{\psi}_{\mu k} \gamma^\mu \Lambda_l X^{klIJ} - \frac{2}{3} \Phi_{(I)}^* \bar{\psi}_{\mu k} \sigma^{\mu\nu} \psi_{\nu l} X^{klIJ} \left. \right\} \\ & + \frac{1}{4} \epsilon_{\alpha\beta} \bar{\psi}_\mu^k \gamma^\mu \phi^\alpha \not{D} \phi^\beta \Lambda_k + \text{h.c.} \\ & + \text{four-fermion terms.} \end{aligned} \quad (3.8)$$

For convenience we have introduced the tensors

$$W_i^{j\dot{I}} = f_{KL}^I \phi_{ik}^K \phi^{kjL}, \quad (3.9)$$

$$X_{ij}^{IJ} = \phi^{klI} f_{KL}^J \phi_{ik}^K \phi_{lj}^L, \quad (3.10)$$

which transform under the 15- and 10-dimensional representations of $SU(4)$, respectively. Derivatives are now also covariant with respect to Yang–Mills transformations of the group G . The four-fermion terms also require the replacement (3.6), but are otherwise the same as in the abelian case.

The investigation of the properties of (3.8) requires the elimination of the auxiliary fields E_{kb} , $T_{ab}{}^{ij}$ and $V_{\mu j}{}^i$. The appropriate equations of motion are:

$$E_{ij} = \eta_{IJ} \{ \bar{\psi}_i^I \psi_j^J - \frac{4}{3} \Phi_{(I)}^* X_{ij}^{IJ} \}, \quad (3.11)$$

$$\eta_{IJ} \left\{ \frac{\Phi_{(I)}^*}{\Phi_{(I)}} K_{\mu\nu}^{+I} \phi_{ij}^J + \hat{F}_{\mu\nu}^{+I} \frac{1}{\Phi_{(I)}} \phi_{ij}^J + \frac{1}{2} \bar{\psi}_i^I \sigma_{\mu\nu} \psi_j^J \right\} = 0, \quad (3.12)$$

$$V_{\mu j}{}^i = \eta_{IJ} \{ A_{\mu}{}^I W_j^J + \frac{1}{2} \phi^{ikl} \bar{\partial}_{\mu} \phi_{jk}^J - \frac{1}{2} \bar{\psi}^I \gamma_{\mu} \psi_j^J \} - \frac{1}{4} \bar{A}^i \gamma_{\mu} A_j - \text{traces} = 0, \quad (3.13)$$

where the dependence on $T_{ab}{}^{ij}$ in (3.12) is given in (3.5). With the dependence of the $\Phi_{(I)}$ on the angles α_I it is no longer possible to obtain a simple algebraic form for $T_{ab}{}^{ij}$.

Substituting (3.11) we find the scalar potential V :

$$V = \frac{4}{9} |\eta_{IJ} \Phi_{(I)}^* X_{kl}^{IJ}|^2 - \frac{1}{2} \eta_{IJ} |\Phi_{(I)}|^2 W_i^I W_j^J. \quad (3.14)$$

It is often convenient to express the scalar fields ϕ_{ij}^I in terms of real fields x_a^I , $a = 1, \dots, 6$, which transform under $SO(6)$. This is done by ($m = 1, 2, 3$)

$$\phi_{ij}^I = \beta_{ij}^m x_m^I + i \alpha_{ij}^m x_{m+3}^I, \quad (3.15)$$

where α^m and β^m are real, antisymmetric 4×4 matrices, satisfying

$$\begin{aligned} [\alpha^m, \alpha^n] &= 2\varepsilon^{mnp} \alpha^p, & [\beta^m, \beta^n] &= 2\varepsilon^{mnp} \beta^p, \\ [\alpha^m, \beta^n] &= 0, \\ \{\beta^m, \beta^n\} &= \{\alpha^m, \alpha^n\} = -2\delta^{mn} \mathbb{1}, \\ \alpha_{ij}^m &= \frac{1}{2} \varepsilon_{ijkl} \alpha_{kl}^m, & \beta_{ij}^m &= -\frac{1}{2} \varepsilon_{ijkl} \beta_{kl}^m. \end{aligned} \quad (3.16)$$

Explicit representations can be found in the literature, e.g. [7]. The constraint (3.2) on the scalar fields takes on the form

$$\eta_{IJ} x_a^I x_b^J = -\frac{1}{4} \delta_{ab}. \quad (3.17)$$

The potential (3.14) can also be expressed in terms of the fields x_a^I . Using (3.9)–(3.10), and the properties (3.16), one finds

$$\begin{aligned} V &= 4 \Phi_{(I)} \Phi_{(J)}^* f_{KL}^I f_{MN}^J x_a^K x_b^L x_a^M x_b^N (\eta_{IJ} + \frac{8}{3} \eta_{IP} \eta_{JQ} x_c^P x_c^Q) \\ &\quad - \frac{8}{9} i (\Phi_{(I)}^* \Phi_{(K)} - \Phi_{(I)} \Phi_{(K)}^*) \eta_{IJ} \eta_{KL} f_{MN}^J f_{PQ}^L \varepsilon^{abcdef} x_a^I x_b^M x_c^N x_d^K x_e^P x_f^Q. \end{aligned} \quad (3.18)$$

In sect. 4 we will consider a number of examples with $P > 6$.

Let us now illustrate (3.8) and (3.18) by considering all possible theories with $P = 6$ and gauged $SU(2) \times SU(2)$. Thus we will recover the known results of [10–12].

As we mentioned above, this means that two angles, α_1 and α_2 , one for each SU(2), can be chosen arbitrarily. Also we will make two coupling constants g_1 and g_2 explicit, by choosing f_{IJ}^K as $g_1 \varepsilon_{IJK456}$ for $I, J, K \leq 3$, and as $g_2 \varepsilon_{123IJK}$ for $I, J, K > 3$. For the case $P=6$ (3.17) is solved by

$$x_a^I = \frac{1}{2} \delta_a^I. \quad (3.19)$$

A short calculation shows that V is given by

$$V = -\frac{1}{2}(g_1^2 |\Phi_1|^2 + g_2^2 |\Phi_2|^2) - i g_1 g_2 (\Phi_1^* \Phi_2 - \Phi_2^* \Phi_1), \quad (3.20)$$

where

$$\Phi_i = e^{i\alpha_i} \phi^1 + e^{-i\alpha_i} \phi^2. \quad (3.21)$$

It is now convenient to solve the constraint (2.10) by

$$\phi_1 = \frac{e^{i\beta}}{\sqrt{1-|Z|^2}}, \quad \phi_2 = \frac{Z e^{-i(\alpha_1+\alpha_2)}}{\sqrt{1-|Z|^2}}, \quad (3.22)$$

where ($\alpha \equiv \alpha_1 - \alpha_2$)

$$e^{i\beta} = \frac{e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2}{|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2|}. \quad (3.23)$$

With this choice (3.20) reads

$$V = -\frac{1}{2} \frac{1}{1-|Z|^2} \{ (g_1^2 + g_2^2)(1+|Z|^2) - 2|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2| \operatorname{Re} Z \} - 2g_1 g_2 \sin \alpha. \quad (3.24)$$

For $\sin \alpha = 0$ this potential takes on the Freedman-Schwarz form [10], and there are no stationary points. For $\sin \alpha \neq 0$ there is a stationary point with $|Z| < 1$ for real Z :

$$Z_0 = \frac{g_1^2 + g_2^2 - 2|g_1 g_2 \sin \alpha|}{|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2|}. \quad (3.25)$$

To determine which parameters are independent one should investigate the normalization of the vector kinetic terms [12]. In the present case these can be obtained explicitly by solving (3.12) for T_{ab}^{ij} . The result is ($m=1, 2, 3$):

$$e^{-1} \mathcal{L}_{\text{kin},A} = -\frac{1}{4} (F_{\mu\nu}^+)^m)^2 \frac{1+Z e^{i(\alpha-\beta)}}{1-Z e^{i(\alpha-\beta)}} - \frac{1}{4} (F_{\mu\nu}^+)^{m+3})^2 \frac{1+Z e^{-i(\alpha+\beta)}}{1-Z e^{-i(\alpha+\beta)}} + \text{h.c.} \quad (3.26)$$

This may be evaluated for the stationary point (3.25). We find

$$e^{-1} \mathcal{L}_{\text{kin},A} = -\frac{1}{4} (F_{\mu\nu}^+)^m)^2 \left(\left| \frac{g_1}{g_2 \sin \alpha} \right| + i \cotg \alpha \right) - \frac{1}{4} (F_{\mu\nu}^+)^{m+3})^2 \left(\left| \frac{g_2}{g_1 \sin \alpha} \right| - i \cotg \alpha \right) + \text{h.c.} \quad (3.27)$$

The terms proportional to $\cotg \alpha$ add a total derivative to the action. The kinetic terms (3.27) have canonical normalization after a rescaling of the Yang–Mills fields. This requires also a change in the coupling constants, and we find

$$\begin{aligned} g'_1 &= \text{sgn } g_1 |g_1 g_2 \sin \alpha|^{1/2}, \\ g'_2 &= \text{sgn } g_2 |g_1 g_2 \sin \alpha|^{1/2}. \end{aligned} \quad (3.28)$$

Effectively the theory depends on only one coupling constant (up to a sign), in agreement with [12]. The cosmological constant is readily evaluated from (3.24) and (3.25):

$$V_0 = -|g_1 g_2 \sin \alpha| - 2g_1 g_2 \sin \alpha \quad (3.29)$$

and depends only on the effective coupling constant. For $g_1 g_2 \sin \alpha < 0$ one finds that V_0 is positive, and that supersymmetry is broken [12]. For $\alpha = \frac{1}{2}\pi$, $g_1 = g_2$, we obtain the potential of [11], which corresponds to gauged $N = 4$ supergravity in the $SO(4)$ -symmetric case. Thus we see that indeed for $\alpha = \frac{1}{2}\pi$ the $SO(4)$ -type theories are obtained, a fact already alluded to at the end of sect. 2.

To conclude this section we present the supersymmetry transformation rules for the fields of $N = 4$ supergravity and Yang–Mills matter. They will be given with the auxiliary field E_{ij} eliminated, since this will facilitate the discussion of the super-Higgs effect in the next section. The transformation rules are:

$$\begin{aligned} \delta \phi_\alpha &= -\bar{\varepsilon}^i \Lambda_i \varepsilon_{\alpha\beta} \phi^\beta, \\ \delta \phi^I_{ij} &= \bar{\varepsilon}_{[i} \psi^I_{j]} - \varepsilon_{ijkl} \bar{\varepsilon}^k \psi^{Il}, \\ \delta \Lambda_i &= 2\varepsilon^{\alpha\beta} \phi_\alpha \not{D} \phi_\beta \varepsilon_i - \frac{4}{3} \varepsilon^j \eta_{IJ} \Phi_{(I)}^* X_{ij}^{IJ} + \varepsilon_{ijkl} \sigma_{ab} \varepsilon^j T_{ab}^{kl} + \psi^2 - \text{terms}, \\ \delta \psi_i^I &= -2 \not{D} \phi_{ij}^I \varepsilon^j - \sigma_{ab} \varepsilon_i \left(\frac{1}{\Phi_{(I)}} \hat{F}_{ab}^I + \frac{\Phi_{(I)}^*}{\Phi_{(I)}} T_{abij} \phi^{ijI} \right), \\ &\quad - 2\varepsilon_j \Phi_{(I)}^* W_i{}^{jI} - \frac{4}{3} \varepsilon_k \eta_{KL} \Phi_{(K)}^* (X_{ij}^{KL} \phi^{jkI} + X^{KLjk} \phi_{ij}^I) + \Lambda^2, \Lambda \psi, \psi^2 - \text{terms}, \\ \delta A_\mu^I &= \Phi_{(I)} (\bar{\varepsilon}^i \gamma_\mu \psi_i^I - 2\bar{\varepsilon}^i \psi_\mu^j \phi_{ij}^I + \bar{\varepsilon}_i \gamma_\mu \Lambda_j \phi^{ijI}) + \text{h.c.}, \\ \delta \psi_\mu^I &= 2 \not{D}_\mu \varepsilon^i - \sigma_{ab} \gamma_\mu \varepsilon_j T_{ab}^{ij} - \frac{2}{3} \gamma_\mu \eta_{KL} \Phi_{(K)}^* X^{ijKL} \varepsilon_j - \varepsilon^{ijkl} \bar{\varepsilon}_j \psi_{\mu k} \Lambda_l + \psi^2, \Lambda^2 - \text{terms}, \\ \delta e_\mu^a &= \bar{\varepsilon}^i \gamma_a \psi_{\mu i} + \text{h.c.} \end{aligned} \quad (3.30)$$

One may verify that these transformations preserve the constraints (3.2)–(3.3). This requires the use of the field equations of the remaining auxiliary fields T_{ab}^{ij} and $V_{\mu j}^i$.

4. Properties and examples

In this section we illustrate the general developments of sect. 3 in two examples. To do so we first discuss the restrictions on the Yang–Mills group G . As we saw in sect. 3, the adjoint representation of G must leave the metric η_{IJ} invariant. This will

be the case if for each real simple subgroup H of G , associated with the gauge fields A_μ^I for $I = M, \dots, N \equiv M - 1 + \dim H$, the corresponding part of the metric, i.e. η_{IJ} for $I, J = M, \dots, N$, is a multiple of the Cartan–Killing metric of H . Since η_{IJ} contains an arbitrary number of positive entries, arbitrarily large compact groups may occur. However, η_{IJ} contains only 6 negative entries and this allows only those noncompact groups which have 6 or less compact generators, or 6 or less noncompact generators. The 11 real simple noncompact groups satisfying these restrictions are given in table 1 [16]. In the examples below we couple one additional matter multiplet to $N = 4$ supergravity by starting from $P = 7$ vector multiplets. The largest possible gauge groups are then $SO(3) \times SO(3)$ and $SO(3) \times SO(2, 1)$ of dimension 6.

Before we embark on a discussion of the specific examples some general remarks about supersymmetry breaking and fermion masses are in order. The scalar potential (3.14), (3.18) can trigger spontaneous breakdown of both the Yang–Mills symmetry and supersymmetry. The fermion mass terms in the lagrangian (3.8) are, after elimination of E_{ij} (3.11):

$$e^{-1} \mathcal{L}_{m,f} = -\frac{2}{3} \eta_{IJ} \Phi_{(I)}^* X^{klIJ} \bar{\psi}_{\mu k} \sigma^{\mu\nu} \psi_{\nu l} + \bar{\psi}_{\mu k} \gamma^\mu (\frac{1}{3} \eta_{IJ} \Phi_{(I)} X^{klIJ} \Lambda_l + \eta_{IJ} \Phi_{(I)}^* W_l^{kl} \psi^{IJ}) \\ + \frac{1}{3} \eta_{IJ} \Phi_{(I)} X^{klIJ} \eta_{KL} \bar{\psi}_k^K \psi_l^L - \eta_{IJ} \Phi_{(I)} \phi^{klJ} f_{KL}^J \bar{\psi}_k^K \psi_l^L + \eta_{IJ} \Phi_{(I)} \bar{\Lambda}_k W_l^{kl} \psi^{IJ} + \text{h.c.}, \quad (4.1)$$

where W_i^{IJ} and X_{ij}^{IJ} are given by (3.9) and (3.10). In the absence of fermion condensation the parts of the transformation rules (3.30) that are relevant for a discussion of supersymmetry breaking, are:

$$\delta \Lambda_i = -\frac{4}{3} \eta_{IJ} \Phi_{(I)}^* X_{ij}^{IJ} \epsilon^j, \quad (4.2)$$

$$\delta \psi_i^I = -2 \epsilon_j (\Phi_{(I)}^* W_i^{jI} + \frac{2}{3} \eta_{KL} \Phi_{(K)}^* (X_{ik}^{KL} \phi^{klI} + X^{klKL} \phi_{kl}^I)), \quad (4.3)$$

$$\delta \psi_\mu^i = -\frac{2}{3} \gamma_\mu \eta_{IJ} \Phi_{(I)}^* X^{ijIJ} \epsilon_j. \quad (4.4)$$

TABLE 1
Simple noncompact gauge groups that may occur in gauged $N = 4$
matter coupling

Group	Dimension	Compact	Noncompact
$SO(2, 1)$	3	1	2
$SO(3, 1)$	6	3	3
$S\ell(3, \mathbb{R})$	8	3	5
$SU(2, 1)$	8	4	4
$SO(3, 2)$	10	4	6
$SO(4, 1)$	10	6	4
$G_{2(2)}$	14	6	8
$SO(3, 3)$	15	6	9
$SU(3, 1)$	15	9	6
$SO(5, 1)$	15	10	5
$SO(6, 1)$	21	15	6

Supersymmetry is spontaneously broken if δA_i or $\delta\psi_i^I$ acquire a vacuum expectation value. The associated massless Goldstone fermion(s) η^k are the combinations that couple to the gravitinos as $\bar{\psi}_{\mu k}\gamma^\mu\eta^k$ in (4.1). After some algebra one finds that in the vacuum the supersymmetry transformation of η^k is

$$\delta\eta^k = -\varepsilon^j (V_0\delta_j^k + \frac{4}{3}\eta_{IJ}\Phi_{(I)}^* X^{klj} \eta_{KL}\Phi_{(K)} X_{jl}^{KL}), \quad (4.5)$$

where V_0 is the vacuum expectation value of the scalar potential. Note the similarity of (4.5) to the corresponding $N=2$ result [17]. If supersymmetry is broken and the super-Higgs effect takes place, the degrees of freedom of η^k are absorbed by the gravitinos. In the case of unbroken supersymmetry the mass parameter $m_{3/2}$ and the cosmological constant V_0 must be related by $V_0 = -3m_{3/2}^2$ [18]. Violation of this relation implies broken supersymmetry, so that supersymmetry breaking can also be inferred from a knowledge of V_0 and $\eta_{IJ}\Phi_{(I)}^* X^{ijIJ}$. This can be seen by inspection of the gravitino mass term in (4.1), or alternatively from the transformation rule (4.4), which should have the form $\delta\psi_\mu = (-\frac{1}{3}V_0)^{1/2}\gamma_\mu\varepsilon$ in the unbroken case.

With regard to the analysis of the spin- $\frac{1}{2}$ mass matrix we note that it is necessary to eliminate the gravitino-goldstino coupling in (4.1). This can be done by performing a shift of the gravitino:

$$\psi_{\mu i} \rightarrow \psi_{\mu i} + \gamma_\mu \varepsilon_i, \quad (4.6)$$

where ε_i solves

$$2\eta_{IJ}\Phi_{(I)}^* X^{klj} \varepsilon_l = \eta^k. \quad (4.7)$$

Furthermore, a normalization of the kinetic term for A_i is required as well: $A_i = \sqrt{2}\tilde{A}_i$.

For $P=6+n$ multiplets the scalar fields ϕ_{ij}^I parametrize the manifold $SO(6, n)/SO(6) \times SO(n)$, which follows from the constraint (3.17) and local $SO(6)$ invariance. Thus ϕ_{ij}^I contributes only $6n$ physical scalar fields in the Poincaré theory. Similarly, only $4n$ of the spin- $\frac{1}{2}$ fields ψ_k^I are independent because of (3.3). To present examples of the results of sect. 3 in which physical properties can be explicitly demonstrated, one needs solutions of the constraints. In this paper we limit ourselves to the case $P=7$, for which a simple solution can be obtained. The constraint (3.17) for the real fields x_a^I is solved by

$$x_a^7 = \frac{1}{2}v_a, \\ x_a^I = \frac{1}{2}\left(\delta_a^I + \frac{v_a v_I}{1 + \sqrt{1 + v^2}}\right), \quad v^2 = v_a v_a, \quad a, I = 1, \dots, 6. \quad (4.8)$$

The scalars v_a parametrize ϕ_{ij}^I by (3.15). In terms of x_a^I (3.3) reads

$$\eta_{IJ}x_a^I\psi_k^J = 0 \quad (4.9)$$

and expresses ψ_k^J for $J=1, \dots, 6$ in terms of the four unconstrained spinors ψ_k^7

and the scalars v_a :

$$\psi_k^J = \frac{v_J}{\sqrt{1+v^2}} \psi_k^7. \quad (4.10)$$

In the first example we consider the case of $P=7$ vector multiplets with gauge group $\text{SO}(3) \times \text{SO}(3)$. Structure constants f_{IJ}^K and $\text{SU}(1,1)$ angles α_I are defined as in sect. 3, where the case $P=6$ was discussed. Without loss of generality, the remaining angle α_7 can be taken equal to zero. Substituting the solution (4.8) of the scalar constraint one finds the potential:

$$V = -\frac{1}{2}(g_1^2|\Phi_1|^2 + g_2^2|\Phi_2|^2) - ig_1g_2\sqrt{1+v^2}(\Phi_1^*\Phi_2 - \Phi_2^*\Phi_1). \quad (4.11)$$

Comparing this result with (3.20) one sees that it has the same stationary points in ϕ_a , supplemented by $v_a = 0$. The discussion of the normalization of the vector kinetic terms and the cosmological constant is therefore also applicable in the present case. In the stationary point one finds

$$\begin{aligned} \eta_{IJ}\Phi_{(I)}X^{klIJ} &= -\frac{3}{4}(g_1\Phi_1 - ig_2\Phi_2)\delta^{kl}, \\ \eta_{IJ}\Phi_{(I)}^*X^{klIJ} &= -\frac{3}{4}(g_1\Phi_1^* - ig_2\Phi_2^*)\delta^{kl}, \end{aligned} \quad (4.12)$$

so that the transformation rules (4.2)–(4.4) become

$$\begin{aligned} \delta A_i &= (g_1\Phi_1^* + ig_2\Phi_2^*)\epsilon^i, \\ \delta\psi_i^7 &= 0, \\ \delta\psi_\mu^i &= \frac{1}{2}(g_1\Phi_1^* - ig_2\Phi_2^*)\gamma_\mu\epsilon_i. \end{aligned} \quad (4.13)$$

Using (3.21)–(3.25) one obtains

$$\begin{aligned} |g_1\Phi_1^* + ig_2\Phi_2^*| &= \sqrt{|g|}(1 - \text{sgn } g), \\ |g_1\Phi_1^* - ig_2\Phi_2^*| &= \sqrt{|g|}(1 + \text{sgn } g), \end{aligned} \quad (4.14)$$

where $g = g_1g_2 \sin(\alpha_1 - \alpha_2)$. Therefore all four supersymmetries are broken if $g < 0$ and unbroken for $g > 0$. This can also be seen from the gravitino mass term in (4.1):

$$\frac{1}{2}(g_1\Phi_1^* - ig_2\Phi_2^*)\bar{\psi}_{\mu k}\sigma^{\mu\nu}\psi_{\nu k} + \text{h.c.}, \quad (4.15)$$

which yields a mass $\frac{1}{2}\sqrt{|g|}(1 + \text{sgn } g)$, compared to the cosmological constant (3.29):

$$V_0 = -g(2 + \text{sgn } g). \quad (4.16)$$

The Yang–Mills symmetry is unbroken in both cases. Of course the above discussion of supersymmetry breaking is directly applicable to the case $P=6$ of sect. 3, and reproduces the known results [10–12].

In the second example we consider again 7 multiplets, now with the gauge group $SO(3) \times SO(2, 1)$. The structure constants are

$$\begin{aligned} SO(3): \quad f_{IJ}^K &= g_1 \varepsilon_{IJK}, & I, J, K &= 1, 2, 3, \\ SO(2, 1): \quad f_{IJ}^K &= g_2 \varepsilon_{1234IJM} \eta^{MK}, & I, J, K &= 5, 6, 7. \end{aligned} \quad (4.17)$$

The abelian factor is associated with the index value $I=4$. We choose $SU(1, 1)$ angles α_1 and α_2 for the $SO(3)$ and $SO(2, 1)$ groups, respectively, and $\alpha_4=0$ for the abelian factor. The solution (4.8) of the scalar constraint equation yields the potential

$$V = -\frac{1}{2}(g_1^2 |\Phi_1|^2 - g_2^2 |\Phi_2|^2) - ig_1 g_2 v_4 (\Phi_1^* \Phi_2 - \Phi_2^* \Phi_1). \quad (4.18)$$

This is stationary in the v_4 -variable only if the last term vanishes, which implies $g_1 g_2 \sin(\alpha_1 - \alpha_2) = 0$. In that case the remainder of the potential has the form

$$V = \frac{1}{2}(g_2^2 - g_1^2) |\Phi|^2, \quad (4.19)$$

which is proportional to the Freedman-Schwarz potential [10]. Therefore there are no stationary points unless the potential vanishes: $g_1^2 = g_2^2$.

Thus we find that flat scalar potentials can also be realized in $N=4$ supergravity. The vacuum expectation value of the scalar fields is undetermined, but it is nevertheless possible to investigate supersymmetry breaking. We choose $g \equiv g_1 = g_2$, $\alpha_1 = \alpha_2 = 0$ (other choices lead to the same conclusions), and consider first the “trivial” vacuum:

$$v_a = 0, \quad \phi_1 = 1, \quad \phi_2 = 0. \quad (4.20)$$

In this case one finds

$$\begin{aligned} \eta_{IJ} \Phi_{(I)}^* X^{klIJ} &= \eta_{IJ} \Phi_{(I)}^* X_{kl}^{IJ} = -\frac{3}{4} g \delta_{kl}, \\ W_i^{jI} &= \begin{cases} \frac{1}{2} g \beta_{ij}^I, & (I=1, 2, 3), \\ 0, & (I=4, 5, 6), \\ -\frac{1}{2} g \alpha_{ij}^1, & (I=7). \end{cases} \end{aligned} \quad (4.21)$$

The transformation rules of the fermions read (4.2)-(4.4)

$$\begin{aligned} \delta \Lambda_i &= g \varepsilon^i, \\ \delta \psi_i^7 &= g \alpha_{ij}^1 \varepsilon_j, \\ \delta \psi_\mu^i &= \frac{1}{2} g \gamma_\mu \varepsilon_i. \end{aligned} \quad (4.22)$$

All four supersymmetries are broken, as one can see from the spin- $\frac{1}{2}$ transformation rules, and also from the gravitino variation ($V_0=0$). The mass of the gravitinos is $m_{3/2} = \frac{1}{2}|g|$. The spin- $\frac{1}{2}$ mass matrix for $\tilde{\Lambda}_i$ ($\sqrt{2}\tilde{\Lambda}_i = \Lambda_i$) and ψ_i^7 reads

$$M_{1/2} = -\frac{1}{6} g \begin{pmatrix} 2\delta_{ij} & \sqrt{2}\alpha_{ij}^1 \\ -\sqrt{2}\alpha_{ij}^1 & \delta_{ij} \end{pmatrix}. \quad (4.23)$$

One verifies that the Goldstone fermions, which take on the form

$$\eta_k = -\frac{1}{2}g(\sqrt{\frac{1}{2}}\tilde{A}_k - \alpha_{kl}^1\psi^{7l}) \quad (4.24)$$

in the vacuum (4.20), are indeed massless.

The Yang-Mills symmetry is also broken by the choice of the vacuum, since the $SO(2, 1)$ generators corresponding to A_μ^5 and A_μ^6 do not annihilate the vacuum. The masses of the vector fields can be deduced from the Yang-Mills covariant scalar kinetic term, which after elimination of $V_{\mu j}^i$ (3.13), and expressed in terms of the real fields x_a^i , reads

$$\mathcal{L}_{\text{kin},s} = -2(\eta_{IJ} + 4\eta_{IK}\eta_{JL}x_a^Kx_a^L)\mathcal{D}_\mu x_b^I\mathcal{D}_\mu x_b^J. \quad (4.25)$$

In the vacuum (4.20) the two vector fields $A_\mu^{5,6}$ each obtain a mass $\sqrt{2}|g|$. Of course the scalar fields all remain massless. A short calculation shows that the graded trace of the square of the mass matrix equals $24m_{3/2}^2$. Note that in this example the 4 physical spin- $\frac{1}{2}$ fields each have mass $m_{3/2}$. Also in $N=2$ matter couplings it was noted that for flat potentials the spin- $\frac{1}{2}$ fields obtain, on average, the gravitino mass [14].

Instead of choosing the vacuum (4.20), one may leave the values of the scalar fields unspecified, and then repeat the analysis of supersymmetry breaking for this example. We shall refrain from presenting the rather complicated formulas which are then obtained. The main conclusion from this more general analysis is that for any values of the scalar fields all four supersymmetries are broken. The most convenient way to obtain this result is by showing that the matrix occurring in the gravitino supersymmetry transformation (4.4) is always nonsingular in this example.

In $N=2$ supergravity it has been shown that supersymmetry cannot be partially broken if the cosmological constant vanishes [19]. It is clearly of interest to find out whether or not a similar situation always holds in $N=4$ supergravity. An answer to this question, which is important for phenomenological applicability of these $N=4$ theories, requires a detailed analysis of the scalar potential (3.14), (3.18) and of the constraint equation (3.2), (3.17). Work along these lines is currently in progress.

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